Inter (Part-I) 2018

Mathematics	Group-I	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: (16)

(i) Simplify $(-1)^{-21}$.

Ans

$$= (i)^{-21}$$
$$= \frac{1}{(i)^{21}} = \frac{1}{i^{20} i}$$
$$= \frac{1}{(i^2)^{10} i} = \frac{1}{(-1)^{10} i} = \frac{1}{i} \times \frac{i}{i}$$
$$= \frac{i}{i^2} = -i$$

(ii) Express the complex number $(1 + i\sqrt{3})$ in polar form.

Ans

$$(1 + i\sqrt{3})$$

Put $r\cos\theta = 1$; $r\sin\theta = \sqrt{3}$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(1)^2 + (\sqrt{3})^2}$$
$$r = \sqrt{1 + 3} = \sqrt{4} = 2$$

Now $\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{3}}{1}$

$$\tan\theta = \sqrt{3}$$
$$\theta = \tan^{-1}\sqrt{3}$$
$$\theta = 60^\circ$$

So $1 + i\sqrt{3} = 2\cos 60^\circ + i\,2\sin 60^\circ$

(iii) Find the multiplicative inverse of $(-4, 7)$.

Ans

$$M.I = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$$

$\Rightarrow$ Here $a = -4$, $b = 7$

$$= \frac{-4}{(-4)^2 + (7)^2}, \frac{-7}{(-4)^2 + (7)^2}$$

$$= \frac{-4}{16+49}, \frac{-7}{16+49}$$

$$= \left[\frac{-4}{65}, \frac{-7}{65}\right]$$

(iv) Is there any set which has no proper subset? If so, name that set.

Ans If two sets A and B are completely equal to each other. So, in such case, both sets A = B has no proper subset.

(v) Write the converse and contrapositive of $\sim q \to \sim p$.

Ans

$$\sim q \to \sim p$$

$$\text{Converse} = \sim p \to \sim q$$

$$\text{Contrapositive} = p \to q$$

(vi) For A = {1, 2, 3, 4}, find the relation in A for R = {(x, y) | x + y < 5}, also write the range of R.

Ans A = {1, 2, 3, 4}

$A \times A$ = {1, 2, 3, 4} $\times$ {1, 2, 3, 4}

= {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}

R = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}.

(vii) If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$, $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find the values of a and b.

Ans

$$A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+2(a) & 1(2)+2(b) \\ a(1)+b(a) & a(2)+b(b) \end{bmatrix}$$

$$= \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix}$$

Given, $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$1 + 2a = 0 \quad ; \quad 2 + 2b = 0$$

$$2a = -1 \quad ; \quad 2b = -2$$

$$\boxed{a = \frac{-1}{2}} \quad ; \quad \boxed{b = -1}$$

(viii) **Find the multiplicative inverse of the matrix $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$.**

Ans Let $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = (2i) \times (-i) - (i)(i)$$
$$= -2i^2 - i^2 = -3i^2$$
$$= -3(-1)$$
$$= 3$$
$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$
$$|A| = 3$$
$$A^{-1} = \frac{1}{3}\begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}$$
$$= \begin{bmatrix} \frac{-i}{3} & \frac{-i}{3} \\ \frac{-i}{3} & \frac{2i}{3} \end{bmatrix}$$

(ix) **Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$.**

Ans L.H.S =

Multiply C_1 with x; C_2 with y ; C_3 with Z.

$$= \frac{1}{xyz}\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ xyz & xyz & xyz \end{vmatrix}$$

Taking xyz common from R_3,

$$= \frac{xyz}{xyz}\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

Interchanging R_1 and R_3,

$$(-1)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix}$$

Interchange R_2 and R_3,

$$(-1)(-1)\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = \text{R.H.S}$$

(x) **Solve the equation $x^4 - 6x^2 + 8 = 0$.**

Ans

$$x^4 - 4x^2 - 2x^2 + 8 = 0$$
$$x^2(x^2 - 4) - 2(x^2 - 4) = 0$$
$$(x^2 - 4)(x^2 - 2) = 0$$

$x^2 - 4 = 0$; $x^2 - 2 = 0$

$x^2 = 4$; $x^2 = 2$

$x = \pm 2$; $x = \pm\sqrt{2}$

$$S.S = x = \{\pm 2\}$$

(xi) **Show that $x^3 - y^3 = (x - y)(x - \omega y)(x - \omega^2 y)$, ω is complex cube root of unity.**

Ans

$$\begin{aligned} R.H.S &= (x - y)(x - \omega y)(x - \omega^2 y) \\ &= (x - y)(x^2 - \omega^2 xy - \omega xy + \omega^3 y^2) \\ &= (x - y)(x^2 - xy(\omega^2 + \omega) + y^2) \\ &= (x - y)(x^2 - xy(-1) + y^2) \\ &= (x - y)(x^2 + xy + y^2) \\ &= x^3 - y^3 \end{aligned}$$

(xii) **If α, β are the roots of $3x^2 - 2x + 4 = 0$, then find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$.**

Ans

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$$

$$\alpha\beta = \frac{c}{a} = \frac{4}{3}$$

$$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\beta^3 + \alpha^3}{\alpha^3\beta^3}$$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^3}$$

$$= \frac{\left(\frac{2}{3}\right)^3 - 3\left(\frac{4}{3}\right)\left(\frac{2}{3}\right)}{\left(\frac{4}{3}\right)^3}$$

$$= \frac{\frac{8}{27} - 4\left(\frac{2}{3}\right)}{\frac{64}{27}}$$

$$= \frac{\frac{8}{27} - \frac{8}{3}}{\frac{64}{27}}$$

$$= \frac{\frac{8-72}{27}}{\frac{64}{27}} = \frac{8-72}{64}$$

$$= \frac{-64}{64} = -1$$

3. Write short answers to any EIGHT (8) questions: (16)

(i) Resolve $\frac{x^2+1}{(x+1)(x-1)}$ into partial fractions.

Ans

$$\frac{x^2+1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \quad \text{(i)}$$

Multiplying by L.C.M $(x+1)(x-1)$ on both sides,

$$x^2 + 1 = A(x-1) + B(x+1)$$

Take $x + 1 = 0 \Rightarrow x = -1$

$$(-1)^2 + 1 = A(-1-1) + B(-1+1)$$

$$1 + 1 = A(-2) + (0)$$

$$2 = -2A$$

$$\boxed{A = -1}$$

Take $x - 1 = 0 \Rightarrow x = 1$

$$(1)^2 + (1) = A(1-1) + B(1+1)$$

$$1 + 1 = A(0) + B(2)$$

$$2 = B(2)$$

$$\boxed{B = 1}$$

Now equation (i) becomes,

$$\frac{x^2+1}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{1}{x-1}$$

(ii) If $a_{n-2} = 3n - 11$, find the nth term of the sequence.

Ans

$$a_{n-2} = 3n - 11$$

Put $n = 3, 4, 5 \ldots$

For $n = 3$

$$a_{3-2} = 3(3) - 11$$

$$a_1 = 9 - 11$$

$$a_1 = -2$$

For $n = 4$

$$a_{4-2} = 3(4) - 11$$

$$a_2 = 12 - 11$$

$$a_2 = 1$$

For $n = 5$

$$a_{5-2} = 3(5) - 11$$
$$= 15 - 11$$
$$a_3 = 4$$

For $n = 6$

$$a_{6-2} = 3(6) - 11$$
$$a_4 = 18 - 11$$
$$a_4 = 7$$

Common difference $d = 7 - 4 = 3$

$$a_4 = a_1 + (n-1)d$$
$$= -2 + (n-1)3$$
$$= -2 + 3n - 3$$
$$a_n = 3n - 5$$

(iii) If 5, 8 are two A.Ms between a and b, find a and b.

Ans Given 5, 8 are two A.Ms between a and b

a, 5, 8, b

$A_1 = a + d_1 \rightarrow$ (i) $\quad A = a + d_2 \quad$ (ii)

where $A_1 = 5$; $A_2 = 8$, $d_1 = 5 - a$, & $d_2 = b - 8$

Put values in equations (i) and (ii), we get

$$5 = a + (5 - a) \quad \text{(iii)}$$
$$5 = a + (b - 8) \quad \text{(iv)}$$

Subtracting equation (iii) and (iv),

$$5 - a - (b - 8) = 0$$
$$5 - a - b + 8 = 0$$
$$-a - b + 13 = 0$$

or $\quad a + b - 13 = 0 \quad$ (v)

$$A_2 = A_1 + d$$
$$8 = 5 + (b - 8)$$
$$8 = b - 3$$
$$b = 8 + 3 \Rightarrow \boxed{b = 11}$$

Put $b = 11$ in equation (v),

$$a + 11 - 13 = 0$$
$$a = 13 - 11$$
$$\boxed{a = 2}$$

(iv) Which term of the A.P. 5, 2, –1, ----- is –85?

Ans Given, AP . 5, 2, –1, - - - , –85

Here $a = 5$,

$d = 2 - 5 = -3$

$a_n = -85$

$n = ?$

$$a_n = a + (n-1)d$$
$$-85 = 5 + (n-1)(-3)$$
$$-85 = 5 - 3n + 3$$
$$-85 = 8 - 3n$$
$$3n = 8 + 85$$
$$3n = 93$$
$$n = \frac{93}{3}$$
$$n = 31$$

Thus $a_{31} = -85$

(v) Insert two G.Ms between 1 and 8.

Ans Let, G_1, G_2 be the two geometric means (G.M's) between 1 and 8.

So, 1, G_1, G_2, 8 are in G.P

Here, $a = 1$,

$n = 4$,

$a_4 = 8$

We know that

$$a_n = ar^{n-1}$$

For $n = 4$

$$a_4 = ar^{4-1}$$
$$8 = 1(r)^3$$
$$(2)^3 = (r)^3$$

$\Rightarrow$ $r = 2$

Therefore,

$$G_1 = ar = (1)(2) = 2$$
$$G_2 = ar^2 = (1)(2)^2 = 4$$

So, the two G.M's between 1 and 8 are : 2, 4.

(vi) If 5 is the harmonic mean between 2 and b, find b.

Ans Here, $a = 2$, $b = b$

We know that

$$H.M = \frac{2ab}{a+b}$$

By given condition,

$\Rightarrow \quad H.M = \frac{2(2)(b)}{2+b} = 5$

$\Rightarrow \quad \frac{4b}{2+b} = 5$

$4b = 5(2+b)$

$4b = 10 + 5b$

$4b - 5b = 10$

$-b = 10$

$\boxed{b = -10}$

(vii) Define fundamental principle of counting.

Ans Suppose A and B are two events. The first event 'A' can occur in p different ways. After, A has occurred, B can occur in q different ways. The number of ways that the two events can occur is the product p . q.

(viii) Find the number of the diagonals of a 6-sided figure.

Ans Number of diagonals

${}^{6}C_2 - 6$

$= 15 - 6 = 9$

(ix) What is probability that a slip of numbers divisible by 4 are picked from the slips bearing number 1, 2, 3, ----, 10?

Ans $S = \{1, 2, 3, ----, 10\}$

$\Rightarrow \quad n(S) = 10$

Let E be the event of picking slip with number divisible by 4.

$E = \{4, 8\}$

$\Rightarrow \quad n(E) = 2$

$\therefore \quad P(E) = \frac{n(E)}{n(S)}$

$= \frac{2}{10} = \frac{1}{5}$

(x) State the principle of mathematical induction.

Ans If a statement S(n) for each positive integer n is such that

(a) S(1) is true; S(n) is true for n = 1.

(b) S(k + 1) is true whenever S(k) is true for any positive integer k, then S(n) is true for all positive integers.

(xi) If x is so small that its square and higher powers can be neglected, then show that:

$$\frac{1-x}{\sqrt{1+x}} = 1 - \frac{3}{2}x$$

Ans L.H.S $= \frac{1-x}{\sqrt{1+x}}$

$= (1-x)(1+x)^{-1/2}$

$= (1-x)(1-\frac{1}{2}x+\frac{3}{8}x^2 - - - - - -)$

$= (1-x)(1-\frac{x}{2})$ by given condition

$= 1-x-\frac{x}{2}+\frac{x^2}{2}$ (Neglected x^2)

$= 1-\frac{3}{2}x = \text{R.H.S}$

(xii) Find the 6th term in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}$.

Ans Let T_{r+1} term in value x.

$T_{r+1} = \binom{10}{r}(x^2)^{10-r}\left(-\frac{3}{2x}\right)^r$

$= \binom{10}{r}x^{20-2r}\left(\frac{-3}{2}\right)^r x^{-r}$

$= \binom{10}{r}\left(-\frac{3}{2}\right)^r x^{20-3r}$

Let: $20-3r = 5$

$r = 5$

$T_6 = \binom{10}{5}\left(\frac{-3}{2}\right)^5 x^5$

$= \frac{-15309}{8}x^5$

Co-efficient $= \frac{-15309}{8}$

4. Write short answers to any NINE (9) questions: (18)

(i) An arc subtends an angle of 70° at the center of a circle and its length is 132 m. Find the radius of the circle.

Ans $\theta = 70°$

In rad $\theta = 70 \times \frac{\pi}{180}$

$= \frac{70}{180} \times \frac{22}{7}$

$$= \frac{11}{9} \text{ rad}$$

$$l = 132 \text{ m}$$

$$r = ?$$

$$\theta = \frac{l}{r}$$

$$r = \frac{l}{\theta}$$

$$= \frac{132}{\frac{11}{9}}$$

$$= 132 \times \frac{9}{11} = 108 \text{ m}$$

(ii) Define coterminal angles.

Ans There are many angles with the same initial and terminal sides. These are called coterminal angles.

(iii) Verify $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$.

Ans $\text{L.H.S} = \sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4}$

$$= \sin^2 30^\circ + \sin^2 60^\circ + \tan^2 45^\circ$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$$

$$= \frac{1}{4} + \frac{3}{4} + 1$$

$$= \frac{1+3+4}{4} = \frac{8}{4} = 2 = \text{R.H.S}$$

(iv) If α, β, γ are angles of a triangle ΔABC, then prove that $\tan(\alpha + \beta) + \tan\gamma = 0$.

Ans As, $\alpha + \beta + \gamma = 180$

$$\alpha + \beta = 180 - \gamma$$

$$\tan(\alpha + \beta) = \tan(180 - \gamma)$$

$$\tan(\alpha + \beta) = -\tan\gamma$$

$$\tan(\alpha + \beta) + \tan\gamma = 0$$

Hence proved.

(v) Find the value of $\sin 105^\circ$, without calculator.

Ans

$$\sin(150)^\circ$$

$$= \sin(45^\circ + 60^\circ)$$

$$= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

(vi) **Prove that $\cot \alpha - \tan \alpha = 2 \cot 2\alpha$.**

Ans R.H.S $= 2 \cot 2\alpha$

$$= \frac{2}{\tan 2\alpha}$$

$$= \frac{2}{\frac{2 \tan \alpha}{1 - \tan^2 \alpha}}$$

$$= \frac{2(1 - \tan^2 \alpha)}{2 \tan \alpha}$$

$$= \frac{1 - \tan^2 \alpha}{\tan \alpha}$$

$$= \frac{1}{\tan \alpha} - \tan \alpha$$

$$= \cot \alpha - \tan \alpha = \text{L.H.S}$$

Hence prove L.H.S = R.H.S

(vii) **Write the domain of $y = \sin x$.**

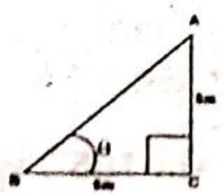
Ans $y = \sin x$

For any real number 'x' has one and only one value from real numbers. So

Domain $= -\infty < x < +\infty$ or set of real numbers.

(viii) **A vertical pole is 8 m high and the length of its shadow is 6 m. What is the angle of elevation of the sun at that moment?**

Ans



From right angle ΔABC,

$$\tan \theta = \frac{8}{6}$$

$$\tan\theta = 1.3333$$
$$\theta = \tan^{-1}(1.3333)$$
$$= 53.13^\circ$$
$$\theta = 53^\circ 8'.$$

(ix) **Find α and β in the triangle ΔABC in which $a = 7$, $b = 7$, $c = 9$.**

Ans $a = 7$, $b = 7$, $c = 9$

$$S = \frac{a+b+c}{2}$$
$$= \frac{7+7+9}{2} = \frac{23}{2} = 11.5$$
$$S - a = 11.5 - 7 = 4.5$$
$$S - b = 11.5 - 7 = 4.5$$
$$S - c = 11.5 - 9 = 2.5$$
$$\tan\frac{\alpha}{2} = \sqrt{\frac{(S-b)(S-c)}{S(S-a)}}$$
$$= \sqrt{\frac{(4.5)(2.5)}{11.5(4.5)}}$$
$$\tan\frac{\alpha}{2} = 0.4601$$
$$\frac{\alpha}{2} = \tan^{-1}(0.4601)$$
$$\alpha = 25^\circ \times 2$$
$$\boxed{\alpha = 50^\circ}$$
$$\tan\frac{\beta}{2} = \sqrt{\frac{(S-c)(S-a)}{S(S-b)}}$$
$$= \sqrt{\frac{(2.5)(4.5)}{11.5\,(4.5)}}$$
$$\frac{\beta}{2} = \tan^{-1}(0.4601)$$
$$\beta = 25^\circ \times 2$$
$$\boxed{\beta = 50^\circ}$$

(x) **Find the area of the triangle ΔABC in which $a = 200$, $b = 120$, $\gamma = 150^\circ$.**

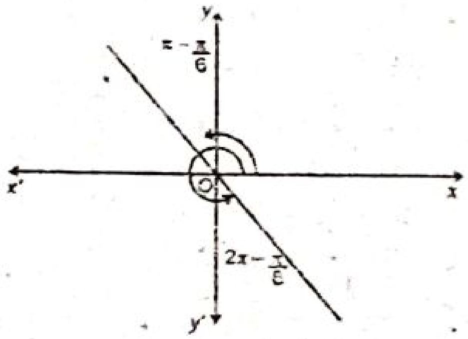
Ans Area of triangle $\Delta = \frac{1}{2} ab \sin\gamma$

$$= \frac{1}{2}(200)(120)\sin 150^\circ$$

$$\Delta = 6000 \text{ sq units}$$

(xi) Evaluate without using calculator $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

Ans $\because$ $\tan\theta$ is (–ve) in II and IV Quadrants with the reference angle $\theta = -\frac{\pi}{6}$.



Therefore $\quad \theta = \pi - \frac{\pi}{6}$ and $2\pi - \frac{\pi}{6}$

$$\theta = \frac{5\pi}{6} \text{ and } \frac{11\pi}{6}$$

(xii) Solve the equation $2\sin x - 1 = 0$.

Ans

$$2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$

$\sin x$ is positive in quadrant I and II with the reference angle $x = \frac{\pi}{6}$.

$$x = \frac{\pi}{6}$$
$$x = \pi - \frac{\pi}{6}$$
$$x = \frac{6\pi - \pi}{6} = \frac{5\pi}{6}$$

Since 2π is the period of $\sin x$

$$S.S = \{\frac{\pi}{6} + 2n\pi,\ \frac{5\pi}{6} + 2n\pi\}$$

(xiii) Find the solution of the equation which lie in interval $[0, 2\pi]$: $\sec x = -2$.

Ans

$$\sec\theta = -2$$
$$\Rightarrow \quad \cos\theta = \frac{-1}{2}$$

$\therefore$ $\cos\theta$ is –ve in second and third quadrants with the angle $\theta = \frac{\pi}{3}$.

$$\therefore \quad \theta = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

and
$$\theta = \pi + \frac{\pi}{3}$$

$$= \frac{4\pi}{3}$$

$$\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \quad \text{Ans.}$$

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Consider the set S = {1, –1, i, –i}. Set up its multiplication table and show that the set S is an abelian group under multiplication. (5)

Ans

⊗	1	–1	i	–i
1	1	–1	i	–i
–1	–1	1	–i	i
i	i	–i	–1	1
–i	–i	i	1	–1

Here $i = \sqrt{-1}$

(i) S is evidently closed w.r.t multiplication.

(ii) It is also associative.

(iii) Identity of S is 1.

(iv) Inverse of each element exists.

As 1 and –1 are inverse of each other.

i and –i are inverse of each other

(v) Multiplication is also commutative.

Hence the given set is an abelion group.

(b) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then find A^{-1} by using adjoint of the matrix. (5)

Ans $A^{-1} = \frac{\text{Adj } A}{|A|}$

$$\text{Cofactors} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 1(2+1) = 3$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = (-1)(0-1) = 1$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = 1(0-2) = -2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = (-1)(0+2) = -2$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1(1-2) = -1$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = (-1)(-1-0) = 1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = 1(0-4) = -4$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = (-1)(1-0) = -1$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 1(2-0) = 2$$

$$\text{Cofactors} = \begin{bmatrix} 3 & 1 & -2 \\ -2 & -1 & 1 \\ -4 & -1 & 2 \end{bmatrix} \quad \text{Adj } A = \begin{bmatrix} 3 & -2 & -4 \\ 1 & -1 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$
$$= 1(2+1) + 0(2-1) + 2(0-2)$$
$$= 1(3) + 0 + 2(-2)$$
$$= 3 - 4$$

$$\boxed{|A| = -1}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{-1}\begin{bmatrix} 3 & -2 & -4 \\ 1 & -1 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

$$= (-1)\begin{bmatrix} 3 & -2 & -4 \\ 1 & -1 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -3 & 2 & 4 \\ -1 & 1 & 1 \\ 2 & -1 & -2 \end{bmatrix}$$

Q.6.(a) Solve the system of equations: $x + y = a + b$; and $\frac{a}{x} + \frac{b}{y} = 2$. (5)

Ans

$x + y = a + b$ $\qquad\qquad$ $\frac{a}{x} + \frac{b}{y} = 2$

$y = a + b - x \quad$ (i) $\qquad$ $ay + xb = 2xy \quad$ (ii)

Put value of y in eq. (ii),

$$a(a + b - x) + bx = 2x(a + b - x)$$
$$a^2 + ab - ax + bx = 2ax + 2bx - 2x^2$$
$$2x^2 - 2bx - 2ax - ax + bx + a^2 + ab = 0$$
$$2x^2 - 1bx - 3ax + a^2 + ab = 0$$
$$2x^2 - x(3a + b) + a^2 + ab = 0$$

Here $a' = 2$; $b' = -(3a + b)$; $c' = a^2 + ab$

$$x = \frac{-b' \pm \sqrt{b'^2 - 4a'c'}}{2a'}$$
$$= \frac{-\{-(3a + b)\} \pm \sqrt{[(3a + b)]^2 - 4(2)(a^2 + ab)}}{2(2)}$$
$$= \frac{3a + b \pm \sqrt{(3a + b)^2 - 8(a^2 + ab)}}{4}$$
$$= \frac{3a + b \pm \sqrt{9a^2 + b^2 + 6ab - 8a^2 - 8ab}}{4}$$
$$= \frac{3a + b \pm \sqrt{a^2 + b^2 - 2ab}}{4}$$
$$= \frac{3a + b \pm \sqrt{(a - b)^2}}{4}$$
$$= \frac{3a + b \pm (a - b)}{4}$$

$x = \frac{3a + b + a - b}{4}$; $\qquad$ $x = \frac{3a + b - a + b}{4}$

$= \frac{4a}{4}$; $\qquad$ $= \frac{2a + 2b}{4}$

$= a$; $\qquad$ $= \frac{2(a + b)}{4}$

$$x = \left(a\,;\,\frac{a+b}{2}\right)$$

When $x = a$:

Put $x = a$ in eq. (i),

$$y = a + b - a$$

$$\boxed{y = b}$$

$$S.S = \{a, b\}$$

When $x = \dfrac{a+b}{2}$

Put $x = \dfrac{a+b}{2}$ in eq. (i),

$$y = a + b - \frac{a+b}{2}$$

$$= \frac{2a + 2b - a - b}{2}$$

$$y = \frac{a+b}{2}$$

$$S.S = \left\{\frac{a+b}{2}, \frac{a+b}{2}\right\}$$

$$\text{Hence } S.S = \left\{a, b, \frac{a-b}{2}, \frac{a+b}{2}\right\}$$

(b) Resolve $\dfrac{9x-7}{(x^2+1)(x+3)}$ into partial fractions. **(5)**

Ans

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1} \quad \text{(i)}$$

Multiply eq. (i) by $(x + 3)(x^2 + 1)$ on both sides,

$$9x - 7 = A(x^2 + 1) + Bx + C\,(x + 3) \quad \text{(ii)}$$

Put $x + 3 = 0 \Rightarrow x = -3$ in eq. (ii),

$$9(-3) - 7 = A((-3)^2 + 1) + Bx + C\,(-3 + 3)$$

$$-27 - 7 = A(9 + 1) + Bx + C\,(0)$$

$$-34 = 10A$$

$$\frac{-34}{10} = A \quad \Rightarrow \quad \boxed{A = \frac{-17}{5}}$$

Now eq. (ii) can be written as

$$9x - 7 = Ax^2 + 1A + Bx^2 + 3Bx + Cx + 3C$$

$$9x - 7 = (A + B)x^2 + (3B + C)\,x + (A + 3C)$$

Compare the coefficients of x^2, x and constants.

$$A + B = 0 \Rightarrow B = -A \Rightarrow \boxed{B = \frac{17}{5}}$$

$$3B + C = 9$$

$$3\left(\frac{17}{5}\right) + C = 9$$

$$\frac{51}{5} + C = 9$$

$$C = 9 - \frac{51}{5}$$

$$= \frac{45 - 51}{5} \Rightarrow \boxed{C = \frac{-6}{5}}$$

Put values of A, B and C in eq. (i),

$$\frac{9x - 7}{(x + 3)(x^2 + 1)} = \frac{-17}{5(x + 3)} + \frac{17x - 6}{5(x^2 + 1)}$$

Q.7.(a) Find four numbers in arithmetic sequence (A.P.) whose sum is 32 and the sum of whose squares is 276. (5)

Ans Let the four terms of A.P are:

$$a + d,\ a - d,\ a + 3d,\ a - 3d$$

By given conditions,

(i) $a + d + a - d + a + 3d + a - 3d = 32$

$$4a = 32 \Rightarrow \boxed{a = 8}$$

(ii) $(a + d)^2 + (a - d)^2 + (a + 3d)^2 + (a - 3d)^2 = 276$

$$a^2 + d^2 + 2ad + a^2 + d^2 - 2ad + a^2 + 9d^2 + 6ad + a^2 + 9d^2 - 6ad = 276$$

$$4a^2 + 20d^2 = 276$$

$$4(8)^2 + 20d^2 = 276$$

$$4(64) + 20d^2 = 276$$

$$256 + 20d^2 = 276$$

$$20d^2 = 276 - 256$$

$$d^2 = \frac{20}{20}$$

$$d^2 = 1 \Rightarrow \boxed{d = \pm 1}$$

Thus $a = 8, d = 1$

$a + d = 8 + 1 = 9$

$a - d = 8 - 1 = 7$

$a + 3d = 8 + 3(1) = 11$

$a - 3d = 8 - 3(1) = 5$
Required numbers = 5, 7, 9, 11.

(b) Use binomial series to show that $1 + \frac{1}{4} + \frac{1 \times 3}{4 \times 8} + \frac{1 \times 3 \times 5}{4 \times 8 \times 12} + ----- = \sqrt{2}$. **(5)**

Ans Let $(1 + x)^n = 1 + \frac{1}{4} + \frac{1 \times 3}{4 \times 8} + \frac{1 \times 3 \times 5}{4 \times 8 \times 12} + ...$

$$1 + nx + \frac{n(n-1)}{2!}x^2 + ... = 1 + \frac{1}{4} + \frac{1 \times 3}{4 \times 8} + \frac{1 \times 3 \times 5}{4 \times 8 \times 12} + ...$$

Comparing both sides, we have

$$nx = \frac{1}{4} \quad \text{(i)} \qquad \Rightarrow \quad n^2x^2 = \frac{1}{16} \qquad \text{(ii)}$$

$$\frac{n(n-1)}{2!}x^2 = \frac{1 \times 3}{4 \times 8}$$

$$n(n-1)\,x^2 = \frac{3}{16} \qquad \text{(iii)}$$

Divide eq. (iii) by eq. (ii),

$$\frac{n(n-1)x^2}{n^2x^2} = \frac{3}{16} \times \frac{16}{1}$$

$$\frac{n-1}{n} = 3$$

$$n - 1 = 3n$$

$$3n - n = -1$$

$$2n = -1 \qquad \Rightarrow \qquad \boxed{n = -\frac{1}{2}}$$

Put value of n in eq. (i),

$$n\,x = \frac{1}{4}$$

$$\left(\frac{-1}{2}\right)x = \frac{1}{4} \qquad \Rightarrow \qquad \boxed{x = \frac{-1}{2}}$$

Put values of n and x in $(1 + x)^n$

$$(1 + x)^n = 1 + \frac{1}{4} + \frac{1 \times 3}{4 \times 8} + \frac{1 \times 3 \times 5}{4 \times 8 \times 12} + ...$$

$$\left[1 + \left(\frac{1}{-2}\right)\right]^{-1/2} = \quad // \quad // \quad //$$

$$\left(\frac{2-1}{2}\right)^{-1/2} = \text{// // //}$$

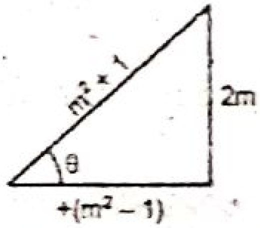
$$\left(\frac{1}{2}\right)^{-1/2} = \text{// // //}$$

$$(2)^{1/2} = \text{// // //}$$

$$\sqrt{2} = 1 + \frac{1}{4} + \frac{1\times 3}{4\times 8} + \ldots$$

Q.8.(a) If $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ and $m > 0$ $\left(0<\theta<\frac{\pi}{2}\right)$, find the values of the all remaining trigonometric ratios. (5)

Ans



$$\operatorname{cosec}\theta = \frac{m^2+1}{2\,m}$$

$$\sin\theta = \frac{2\,m}{m^2+1}$$

$$|AB|^2 = (m^2+1)^2 - (2\,m)^2$$
$$= m^4 + 2\,m^2 + 1 - 4\,m^2$$
$$|AB|^2 = m^4 - 2\,m^2 + 1$$
$$|AB|^2 = (m^2-1)^2$$
$$\sqrt{|AB|^2} = \sqrt{(m^2-1)^2}$$
$$(AB) = m^2 - 1$$

$$\cos\theta = \frac{m^2-1}{m^2+1}$$

$$\sec\theta = \frac{m^2+1}{m^2-1}$$

$$\tan\theta = \frac{2\,m}{m^2-1}$$

$$\cot\theta = \frac{m^2-1}{2\,m}$$

(b) Prove that $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9} = \frac{3}{16}$ without using calculator. (5)

Ans As $\frac{\pi}{9} = \frac{180}{9} = 20^\circ$; $\frac{\pi}{3} = \frac{180}{3} = 60^\circ$

$\frac{2\pi}{9} = \frac{2(180)}{9} = 40^\circ$; $\frac{4\pi}{9} = \frac{4(180)}{9} = 80^\circ$

$$\text{L.H.S} = \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$$

$$= \sin 20^\circ \sin 40^\circ \left(\frac{\sqrt{3}}{2}\right) \sin 80^\circ$$

$$= \frac{\sqrt{3}}{-2 \times 2} \sin 20^\circ (-2 \sin 40^\circ \sin 80^\circ)$$

$$= \frac{\sqrt{3}}{-4} \sin 20^\circ (\cos (80^\circ + 40^\circ) - \cos (80^\circ - 40^\circ))$$

$$= \frac{\sqrt{3}}{-4} \sin 20^\circ (\cos 120^\circ - \cos 40^\circ)$$

$$= \frac{\sqrt{3}}{-4} \sin 20^\circ \left(-\frac{1}{2} - \cos 40^\circ\right)$$

$$= \frac{\sqrt{3}}{-4} \left[\frac{-\sin 20^\circ}{2} - \sin 20^\circ \cos 40^\circ\right]$$

$$= \frac{\sqrt{3}}{-4} \left[\frac{-\sin 20^\circ}{2} - \frac{1}{2}(2 \sin 20^\circ \cos 40^\circ)\right]$$

$$= \frac{\sqrt{3}}{-4} \left[\frac{-\sin 20^\circ}{2} - \frac{1}{2}(\sin (40^\circ + 20^\circ) - \sin (40^\circ - 20^\circ))\right]$$

$$= \frac{\sqrt{3}}{-4} \left[\frac{-\sin 20^\circ}{2} - \frac{1}{2} \sin 60^\circ + \frac{1}{2} \sin 20^\circ\right]$$

$$= \frac{\sqrt{3}}{-4} \left(\frac{-1}{2} \sin 60^\circ\right)$$

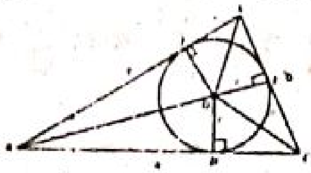
$$= \frac{\sqrt{3}}{-4} \left(\frac{-1}{2} \frac{\sqrt{3}}{2}\right) = \frac{3}{16}$$

Q.9.(a) With usual notations, prove that $r = \frac{\Delta}{s}$. **(5)**

Ans Let a triangle = ABC

Let the internal bisectors of angles of triangle ABC meet at O, the in centre.

Draw $OD \perp BC$; $\overline{OE} \perp \overline{AC}$ and $\overline{OF} \perp \overline{AB}$



Let $m\overline{OD} = \overline{OE} = \overline{OF} = r$

From the Figure:

Area of ΔABC = Area ΔOBC + Area ΔOCA + Area ΔOAB

$$\Delta = \frac{1}{2}(BC \times OD) + \frac{1}{2}(CA \times OE) + \frac{1}{2}(AB \times OF)$$

$$[\because \quad \Delta ABC = \frac{1}{2}(\text{Base} \times \text{Height})]$$

$$= \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$$

$$= \frac{1}{2}r(a + b + c)$$

$$= \frac{1}{2}r(2s) \qquad \because \quad (2s = a + b + c)$$

$$\Delta = rs$$

$$\boxed{\frac{\Delta}{s} = r}$$

Hence proved.

(b) Prove that $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}$. **(5)**

Ans L.H.S

$$\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17}$$

$$= \sin^{-1}\left\{\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2} + \frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right\}$$

$$= \sin^{-1}\left\{\frac{3}{5}\sqrt{\frac{289-64}{289}} + \frac{8}{17}\sqrt{\frac{25-9}{25}}\right\}$$

$$= \sin^{-1}\left\{\frac{3}{5}\sqrt{\frac{225}{289}} + \frac{8}{17}\sqrt{\frac{16}{25}}\right\}$$

$$= \sin^{-1}\left\{\frac{3}{5} \times \frac{15}{17} + \frac{8}{17} \times \frac{4}{5}\right\}$$

$$= \sin^{-1}\left\{\frac{9}{17} + \frac{32}{85}\right\} = \sin^{-1}\left\{\frac{45+32}{85}\right\}$$

$$= \sin^{-1}\left\{\frac{77}{85}\right\} = \text{R.H.S}$$